Govt Arts and Science college for Women

Bargur

I^{st} - cyclic Examination (2018)

Operations Research

Course : III B.Sc Mathematics

Time : 1½ hours

Max. Marks: 50 Marks

Sec - A

Answer all the questions (5×2 =10)

1. Define : OR
2. What is the feasible region
3. Describe the simplex method
4. Define : Slack variable
5. What is LPP.

Sec - B

(4×10 = 40)

Answer any four questions

1. Write down the maximization of $z = 4x_1 + 3x_2$ S.to.C $2x_1 + x_2 \leq 1000$, $x_1 + x_2 \leq 800$, $x_1 \leq 400$, $x_2 \leq 700$ using graphical method find the feasible region.
2. Using the simplex method to solve following LPP. max $z = 5x_1 + 4x_2$ S.to.C $4x_1 + 5x_2 \leq 10$, $3x_1 + 2x_2 \leq 9$, $8x_1 + 3x_2 \leq 12$, and $x_1, x_2 \geq 0$.
3. Max $z = 21x_1 + 15x_2$ S.to.C $-x_1 - 2x_2 \geq -6$, $4x_1 + 3x_2 \leq 12$ and $x_1, x_2 \geq 0$.
4. mini $z = 8x_1 - 2x_2$ S.to.C $-4x_1 + 2x_2 \leq 1$, $5x_1 + 4x_2 \leq 3$, $x_1, x_2 \geq 0$
5. Big - M Method : Min $z = 12x_1 + 20x_2$. S.t.C $6x_1 + 8x_2 \geq 100$ $7x_1 + 12x_2 \geq 120$ and $x_1, x_2 \geq 0$.

K. Balasamy

HOD signature 2/8/18

II - cycle test - (2018) BARGUR

OPERATION RESEARCH

III - B.Sc (Maths)

38 copies

PART A

I WRITE ALL QUESTION.

1. Define Feasible Solution
2. Define Basic Feasible Solution
3. Define optimal Solution
4. Define Non-degenerate Basic feasible Solution.
5. Define Degenerate Basic Feasible Solution

PART B

I. Write all question.

6. a. Find the IBFS using North-West Corner rule

	E	F	G	H	Availability
A	4	8	10	16	100
B	7	2	3	11	200
C	5	9	11	2	300
Demand	160	240	105	95	

(or)

b. Find the IBFS using North-West Corner Rule

	D_1	D_2	D_3	
O_1	5	7	8	70
O_2	4	4	6	30
O_3	6	7	7	60
	65	42	43	

7. a. obtain IBFS using Least cost Method.

	D	E	F	G	
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
	200	225	275	250	

(or)

b. Find IBFS by the least cost method.

	W_1	W_2	W_3	W_4	availability
F_1	48	60	56	58	140
F_2	45	55	53	60	260
F_3	50	65	60	62	360
F_4	52	64	55	61	220
	200	320	250	210	

8. a Find IBFS by VAM availability

	9	12	9	6	9	10	5
	7	3	7	7	5	5	6
	6	5	9	11	3	11	2
	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	

(b)

b. Find IBFS by Vogel's Approximation method

	D_1	D_2	D_3	D_4	capacity
O_1	40	25	22	33	200
O_2	44	35	30	30	60
O_3	38	38	28	30	140
Demand	200	40	120	40	

[illegible] method

7 a. solve the following linear programming problem by ~~simplex form~~

Minimise $z = 16x_1 + 16x_2$

Subject to $2x_1 + 4x_2 \geq 3$

$3x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$

(or)

b. Solve the following LPP by big M method

Maximise $z = x_1 + 2x_2$

Subject to $x_1 - x_2 \geq 3$

$2x_1 + x_2 \leq 10$

$x_1, x_2 \geq 0$

RESEARCH

III B.Sc (Maths)

PART - A (10×2 = 20)

Answer all the Questions:-

1. Write about infeasible solution in LPP.
2. What is an optimal solution?
3. ~~Stat~~ Define linear programming
4. Define surplus variable and artificial variable.
5. What does "VAM" mean?
6. Define meaning of Transportation problem.
7. Define Feasible Solution.
8. Define Assignment problem.
9. State the meaning of degeneracy.
10. Describe the term total elapsed time.

PART - B.

Answer all the Questions:- (5×5 = 25)

11. a. Solve the LPP. max $z = 5x_1 + 3x_2$ subject to $3x_1 + 5x_2 \le 15$, $5x_1 + 2x_2 \le 10$, where $x_1, x_2 \ge 0$ by graphical method. (or)

b) Use simplex method to solve max $z = x_1 + x_2 + 3x_3$ subject to the constrains $3x_1 + 2x_2 + x_3 \le 3$, $2x_1 + x_2 + 2x_3 \le 2$, and $x_1, x_2, x_3 \ge 0$.

12. a. Use Big-M method to solve max $z = 3x_1 + 2x_2 + 3x_3$ subject to $2x_1 + x_2 + x_3 \le 2$, $3x_1 + 4x_2 + 2x_3 \ge 8$; $x_1, x_2, x_3 \ge 0$.

(or)

Solve the LPP by dual simplex method. maximize $Z = -3x_1 - x_2$, subject to $x_1 + x_2 \ge 1$, $2x_1 + 3x_2 \ge 2$, $x_1, x_2 \ge 0$.

Find IBFS to the TP using North-West corner rule

a) From

	E	F	G	H	Availability
A	4	8	10	16	100
B	7	2	3	1	200
C	5	9	11	2	300
Demand	160	240	105	95	

(or) b) Using matrix minimum method

	D	E	F	G	
A	11	13	17	14	250
B	21	24	13	10	300
C	16	18	14	10	400
	200	225	275	250	

4.a. Solve the following minimal assignment problem by hungarian method

Machines

Jobs	1	2	3	4
A	9	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

(or) Unbalance Assignment problem

Jobs	I	II	III	IV	V
A	20	21	14	12	18
B	17	21	20	24	22
C	15	16	19	22	24
D	23	25	21	20	17

15.a.

Job	1	2	3	4	5
Machine A	10	2	18	6	20
Machine B	4	12	14	16	8

Determine the sequence for the Job will minimise the total elapsed time.

(or)

Job	1	2	3	4	5	6	7
turning	8	10	10	6	12	1	3
threading	3	12	15	6	10	11	9

PART-C (3×10 = 30)

1. Solve by Big-M method $\min z = 12x_1 + 18x_2 + 15x_3$ subject to $4x_1 + 8x_2 + 6x_3 \geq 64$, $3x_1 + 6x_2 + 12x_3 \geq 96$, $x_1, x_2, x_3 \geq 0$.
2. Solve by two phase method: $\min z = x_1 + x_2$ subject to $x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$, $x_1, x_2 \geq 0$.
3. Solve by VAM & find IBFS

	1	2	3	4	5	6	Availability
A	9	12	9	6	9	10	5
B	7	3	7	7	5	5	6
C	6	5	9	11	3	11	2
D	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	

4. Solve maximation Assignment problem.

Job	A	B	C	D	E
1	17	23	25	13	25
2	25	9	13	6	21
3	26	12	18	15	22
4	7	23	26	21	31
5	14	18	25	20	24

A. Regina Mary
Incharge.

Govt Arts And Science College for Women-Bargur

REAL ANALYSIS- II Model examination: 2019 Time:3.00 hrs

III-BSc MATHEMATICS Maximum:75 marks

I.Answer all the Questions(10*2=20)

1. Define totally bounded
2. Define ϵ dense set
3. Define Uniformly continuous
4. Define complete metric space
5. Define measure zero of a set
6. Define Reimann Integral
7. State first fundamental theorem of calculus
8. Prove that $\int_1^\infty \frac{1}{x}\,dx$ diverges
9. Define converges of sequence of functions
10. State Dini's theorem for series of functions

II. Answer all the Questions(5*5=25)

11. a)If the subset A of the ‹M,ρ› is totally bounded then prove that is bounded (or)
    b)Show that l^2 is complete
12. a)If the metric space M has the Heine-Borel property the prove that M is compact (or)
    b)Let ‹M_1,ρ_1› be a compact metric space.If f is continuous from M_1 into a metric space ‹M_2,ρ_2›, prove that f is uniformly continuous on M_1
13. a) State and prove Chain Rule for derivatives (or)
    b)If f ε R[a,b] and a<c<b then prove that $\int_a^b f = \int_a^c f + \int_c^b f$
14. a)State and prove law of the mean (or)
    b) State and prove Roll's theorem
15. a)Let $\{fn\}$ be a sequence of function in R[a,b] which converges uniformly to the function f on [a,b] then f ε R[a,b] and $Lt \lim_{n\to\infty} \int_a^b fn(x)\ dx = \int_a^b f(x)$ (or)
    b)Let $\{fn\}$ be a sequence of real valued functios on a metric space M which converges uniformly to the function f on M. If each f_n(n ε I) is continuous at a ε M then prove that f is also continuous at a.

III. Answer any three questions(3*10=30)

16. Prove that the subset of the metric space ‹M,ρ› is totally bounded iff every sequence of points of A contains a cauchy subsequence
17. Prove that the metric space ‹M,ρ› is complete iff every sequence of points in M has a subsequence of points in M has a subsequence converging to a point in M
18. Let f be a bounded function on [a,b],prove that f ε R[a,b] iff for each ε>0,there exists a subdivision ρ of [a,b] such that U[f; ρ]<L[f; ρ]+ ε
19. State and prove the first fundamental theorem of calculus.
20. State and prove Cauchy creterien for Uniform convergence.

SUBJECT INCHARGE

HOD SIGNATURE 11/03/19

I, F1,F2, measure 2020.

Govt Arts and Science College for Women - Bargur.

II - Cyclic Test [Feb. 2019]

Paper Name: Real Analysis - II — Time: 2:00 hrs

Paper Code: 12UMA11 — Maximum: 50 marks.

PART - A

I. Answer All the Questions (5×2=10 marks)

1. Define Riemann integral.
2. If $f(x)=x$ $(0 \le x \le 1)$ and $\sigma: \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ is a subdivision of $[0,1]$, compute $U(f:\sigma)$.
3. Define set of measure zero.
4. Show that the function $f(x)=x^2$ is uniformly continuous on $[0,1]$.
5. Define derivative of a function at a point 'c'.

II. Answer All the Questions (4×5=20 marks)

6. a) If f be a continuous function from a compact metric space M into R', prove that f attains a maximum and minimum values at some point of M. (or)

b) If the metric space M has the Heine-Borel property then prove that M is compact.

b) If $f(x)=x^2$ for all $x \in R'$ S.T f is not uniformly continuous on R'.

7. a) Let $\langle M_1, \rho_1 \rangle$ be a compact metric space. If f is a continuous from M_1 into a metric space $\langle M_2, \rho_2 \rangle$ Prove that f is uniformly continuous on M_1.

b) Let A be a subset of the metric space $\langle M, \rho \rangle$. If $\langle A, \rho \rangle$ is compact then prove that A is closed subset of $\langle M, \rho \rangle$.

8. a) If each of the subsets $E_1, E_2, \dots$ of R is of measure zero, then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

b) State and prove chain rule for derivatives.

9. a) If $f \in R[a,b]$, $g \in R[a,b]$ then prove that $f+g \in R[a,b]$ and $\int_a^b f+g = \int_a^b f + \int_a^b g$. (or)

b) Let f be a bounded function on $[a,b]$. Then every upper sum for f is greater than (or) equal to every lower sum for f. That is if σ & T are any two subdivisions of $[a,b]$ then prove that $U[f:\sigma] \geq L[f:T]$

III Answer Any TWO Questions (2×10=20 marks).

10. Let f be a bounded function on $[a,b]$ prove that $f \in R[a,b]$ iff for each $\epsilon > 0$ there exists a subdivision σ of $[a,b]$ such that $U[f:\sigma] < L[f:\sigma] + \epsilon$.

11. If the metric space M has a Heine-Borel property then M is compact.

12. Let f be a continuous function from the compact metric space M_1 into the metric space M_2 then prove that the range $f(M_1)$ is also compact.

K.N. SUDHA
Subject Incharge:

HOD Signature

III. B.Sc Mathematics - Cyclic Test - I [Jan-2019]

Paper Name : Real Analysis - II | Maximum : 50 marks

Paper Code : 12UMA11 | Time : 01:30 hrs.

PART-A [5×2 = 10 marks]

Answer All the Questions.

1. Define totally bounded
2. Define ε-dense set.
3. Define Compact metric space
4. Define Complete metric space
5. Define Contraction.

PART-B [4×5 = 20 marks]

Answer All the Questions.

6 a) If the subset A of the metric space $\langle M, \rho \rangle$ is totally bounded then prove that is bounded (or)

b) Let M be a metric space then M is connected iff every continuous characteristic function of M is constant.

7 a) Under the continuous function the image of connected sets is connected (or)

b) State and Prove the Generalization of the Nested Interval theorem

8. a) Let A be a subset of the metric space $\langle M, \rho \rangle$. If $\langle A, \rho \rangle$ is compact then A is closed subset of $\langle M, \rho \rangle$. (or)

b) Under continuous image of a compact set is also compact.

9. a) Prove that Union of Connected set is connected. (or)

b) Let $\langle M, \rho \rangle$ be a compact metric space. If A is closed in M, then $\langle A, \rho \rangle$ is also compact.

PART-C [2×10 = 20 marks]

10. Let $\langle M, \rho \rangle$ be a metric space the subset A of M is totally bounded iff every sequence of points of A contains a cauchy subsequence.

11. State and Prove Picard fixed point theorem.

12. The metric space $<M, \rho>$ is compact iff every sequence of point in M has a sequence covering to a point in M.

Subject Incharge

HOD Signature.

Govt Arts & Science College for women, Bargur.

I cyclic Test-Feb-2020

Subject : Graph Theory
Course : II M.Sc(Maths)

Max marks:50
Time:1 1/2hrs

SECTION-A

Answer all the questions: **5*2=10 Marks**

1) Define Graph with examples.
2) Define complete graph.
3) Define cut – edge with example.
4) Define connectivity.
5) Define spanning tree.

SECTION-B

Answer all the question: **2*5=10 Marks**

6) a) If G is simple and $\delta \geq n-1/2$, then G is connected. (or)
   b) In any graph G, the number of vertices of odd degree is even.

7) a) A vertex V of a connected graph G with atleast three vertices is a cut vertex of G, iff there exist vertices u and w of G, distinct from V, such that V is in every u-w path in G. (or)

   b) A simple cubic (3 regular) connected graph G has a cut vertex iff it has a cut edge.

Answer any three questions: **SECTION-C** **3*10=30 Marks**

8. A graph is bipartite if and only if it contains no odd cycles.

9. a) If a simple graph G is not connected. Then G^c is connected.

   b) The number of edges of a simple graph with w components cannot exceed

   (n-w)(n-w+1)/2

10. State and prove Redel theorem.

11. State and prove cayley's formula.

S. Suvathi 22/01/2020
Subject Incharge.

24/01/2020
HOD Signature.

Govt. Arts and science college for Women, Bargur.

II cyclic Test - March-2020.

Sub: Graph Theory | Max. Marks : 50
course: II M.Sc (Maths) | Time : $1\frac{1}{2}$ hrs

Section-A

Answer all questions $\quad 5\times2=10$

1. Define a matching of a graph.
2. Define Eulerian graph.
3. Define a maximal independent set.
4. What is a traceable graph? Give example.
5. Define proper colouring.

Section-B

$2\times5=10$.

6. a) prove that every connected 3-regular graph having no cut edges has a 1-factor.

(or)

b) prove that if G is Hamiltonian, then every non-empty proper subset S of V, $w(G-S)\le|S|$.

7. a) prove that in a critical graph G, no vertex cut is a clique

(or)

b) prove that, if G is a loopless bipartite graph, then, $\chi'(G)=\Delta(G)$.

Section - C

$3 \times 10 = 30$

Answer any three questions.

8. Show that a graph G is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G.

9. State and prove Tutte's 1-factor theorem.

10. State and prove Brook's theorem.

11. State and prove petersen's theorem.

— × —

S. Santhi 27/02/2020
Subject Incharge

27/02/2020
HOD Signature.

Govt Arts and Science College for Women- Bargur.

Department of Mathematics.

II B.Sc., Mathematics

Cyclic Test . I [Feb- 2021]

Sub: Trigonometry and Analytical Geometry of 3D.

Sub Code : 17OMA08.

Max: 50 Marks.

Time : 2.00 hrs.

PART- A

I. Answer all the Questions (5×1=5)

1. Write the binomial Expression for $(x+a)^n$.

2. Write the Demoivre's thm for $(\cos\theta + i\sin\theta)^n$.

3. Write the exponential Series for e^x.

4 Write down the period of hyperbolic sine, hyperbolic cosine and hyperbolic tangent.

5. $\tanh^{-1}x =$ ________

PART- B

II. Answer all the Questions (3×5=15)

6. prove that $\frac{\sin 5\theta}{\sin\theta} = 5 - 20\sin^2\theta + 16\sin^4\theta$.

7 prove that $-64\sin^7\theta = \sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta$

8. prove that $\tanh 3x = \frac{3\tanh x + \tanh^3 x}{1+3\tanh^2 x}$

PART- C

III Answer all the Questions. (3×10=30)

9. prove that $\cos 8\theta = 1 - 32\sin^2\theta + 160\sin^4\theta - 256\sin^6\theta + 128\sin^8\theta$

10. Prove that $64\cos^7\theta = \cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta$

11. If $\tan(\theta + i\varphi) = \cos\alpha + i\sin\alpha$. Show that

(i) $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$

(ii) $\varphi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$

Govt Arts and Science College for women - Bargur.

II Cyclic Test [Mar-2021]

II. B.Sc Mathematics

Paper Name: Trignometry and Analytical Geometry 3D   Time. 2.00 hrs

Paper Code : 17UMA08   Maximum: 50 mark

I. Answer all the Questions. $(5\times2=10)$

1. If $\tan y = \tan\alpha \tanh\beta$, $\tan z = \cot\alpha \tanh\beta$ prove that $\tan(y+z) = \sinh 2\beta \operatorname{cosec} 2\alpha$.

2. If $\tan(\alpha+i\beta) = x+iy$ prove that $x^2+y^2+2x\cot 2\alpha = 1$

3. Seperate into real & imaginary parts of $\tanh(x+iy)$

4. Find the value of $\operatorname{Log}(4+3i)$

5. Prove that $\log \frac{a+ib}{a-ib} = 2i\tan^{-1}\frac{b}{a}$.

II Answer All the Questions. $(4\times5=20)$

6 a) Seperate into real & imaginary parts of $\tan^{-1}(x+iy)$ (or)

b) If $\cos(x+iy) = r(\cos\alpha + i\sin\alpha)$ $y = \frac{1}{2}\log\left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$

7 a) If $\tan(\alpha+i\beta) = i$, α and β being real prove that α is intermed indeterminate and β is infinite. (or)

b) Prove that if $(1+i\tan\alpha)^{1+i\tan\beta}$ can have real values, one of them is $(\sec\alpha)^{\sec^2\beta}$.

8 a) Prove that $\log \frac{\cos(x-iy)}{\cos(x+iy)} = 2i\tan^{-1}(\tan x \tanh y)$ (or)

b) Show that $\log\tan\left(\frac{\pi}{4}+i\frac{x}{2}\right) = i\tan^{-1}(\sinh x)$

9 a) Find the point where the line $\frac{x-2}{2} = \frac{y-4}{-3} = \frac{z+6}{4}$ meets the plane $2x+4y-z-2=0$. (or)

b) Find the image of the point $(1,-2,3)$ in the plane $2x-3y+2z+3=$

III Answer any two Questions, $(2\times10=20)$

10. Find the equations of the image of the line $\frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{2}$ in the plane $2x-3y+2z+3=0$.

11. If $\tan(\theta+i\varphi) = \cos\alpha + i\sin\alpha$ show

i) $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$

ii) $\varphi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$

12. If $\log\sin(\theta+i\varphi) = A+iB$ show that

i) $2e^{2A} = \cosh 2\varphi - \cos 2\theta$

ii) $\cos(\theta - B) = e^{2\varphi}\cos(\theta+B)$

Subject Incharge

HOD Signature 20/12/19

Govt. Arts and Science College for Women, Bargur
Model Examination Feb 2023

Class: I M.Sc (Mathematics) Max Marks: 75
Sub: Real Analysis Time: 3 hrs

Section A

Choose the Correct answer. 12x1=12

1. Every infinite subset of a countable set is ____
a) countable b) uncountable c) Infinite d) finite

2. The set of all rational numbers is ____
a) uncountable b) countable c) finite d) Infinite

3. A finite point set has ____ points a) limit b) no limit c) more

4. Every bounded sequence in R^k contains ____ subsequence
a) divergent b) Convergent c) Infinite d) finite

5. A metric space in which every cauchy sequence converges is ____
a) Incomplete b) complete c) Convergent d) Countable

6. Suppose $\{s_n\}$ is monotonic then $\{s_n\}$ converges if and only if it is ____
a) Convergent b) bounded c) unbounded d) complete

7. If a function f has a limit at p, the limit is ____
a) Infinite b) uncountable c) unique d) more than one

8. If f and g be complex continuous functions on a metric space X, then these are continuous on X
a) $f+g$, f/g b) $f+g$ & fg c) fg, f/g d) $f+g$ & fg

9. Monotonic functions have ____ a) discontinuous of the second kind
b) no discontinuous of second kind c) continuous d) uniformly continuous

10. A continuous function is uniformly continuous on the compact metric space if ____
a) bounded b) closed c) compact d) countable

11. If f is differentiable at a point $x \in [a,b]$, then f is ____
a) bounded at x b) finite at x c) continuous at x
d) None of these.

12. The set of real numbers R is a) uncountable
b) countable c) Infinite d) bounded

13. $\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n}$ equals a) 1 b) 2 c) 3 d) 0

14. The sequence $\langle s_n \rangle$ defined as $s_n = \frac{(-1)^n}{n}$ a) diverges b) converges c) oscillates d) None of these

15. The intersection of any collection of closed sets is a) closed set b) open set c) null set d) None of these

Section-B

Answer any two questions. $2 \times 5 = 10$

16. Prove that, every infinite subset of a countable set A is countable.

17. Suppose $\{s_n\}$ is monotonic. Then $\{s_n\}$ converges if and only if it is bounded.

18. State and prove Leibnitz test.

19. Suppose f is a continuous mapping of a compact metric space X into a metric space Y, then f(X) is compact.

20. If f a real differentiable function on [a,b] and suppose $f'(a) < \lambda < f'(b)$ then there is a pt $x \in (a,b)$ such that $f'(x) = \lambda$

Section-C

Answer all questions. $5 \times 10 = 50$

21. a) Let $\{E_n\}$, $n = 1,2,3 \ldots$ be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$. Prove that S is countable.

(or)

b) Prove that compact subsets of metric space are closed.

22. a) Suppose $\{s_n\}$, $\{t_n\}$ are complex sequences, and $\lim_{n\to\infty} s_n = s$, $\lim_{n\to\infty} t_n = t$. Then, a) $\lim_{n\to\infty}(s_n + t_n) = s + t$ b) $\lim_{n\to\infty} c s_n = cs$, $\lim_{n\to\infty}(c + s_n) = c + s$, for any number c c) $\lim_{n\to\infty} s_n t_n = st$ d) $\lim_{n\to\infty} \frac{1}{s_n} = \frac{1}{s}$, provided $s_n \neq 0$ $(n = 1,2,3 \ldots)$ and $s \neq 0$.

(or)

b) i) If $\{p_n\}$ is a sequence in a complete metric space X. PT some sub-sequence of $\{p_n\}$ converges to a point of X.

ii) Every bounded sequence in R^k contains a convergent subsequence.

23. a) State and prove partial summation formula.

(or)

b) Suppose a) $\sum_{n=0}^{\infty} a_n$ converges absolutely b) $\sum_{n=0}^{\infty} a_n = A$ c) $\sum_{n=0}^{\infty} b_n = B$, d) $C_n = \sum_{k=0}^{n} a_k b_{n-k}$ $(n = 0,1,2 \ldots)$

then $\sum_{n=0}^{\infty} C_n = AB$.

24\. a) A mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

(or)

b) Let f be a continuous mapping of a compact metric space X into a metric space Y, then f is uniformly continuous on X.

25\. a) State and prove chain rule for differentiation.

(or)

b) If f is continuous mapping of $[a,b]$ into R^k and f is differentiable in (a,b), then there exists $x \in (a,b)$ such that $|f(b)-f(a)| \leq (b-a)|f'(x)|$.

S. Sathi...

Subject Incharge

Govt Arts and Science College for Women - Bargur

III - B.Sc Mathematics - Model Exam (2023) : 5 copies

Time: 3hrs Operational Research Max Marks: 75

Part - A

I Answer all the questions $15 \times 1 = 15$

1. A constraint in an LPP is expressed as
(a) an equation with '=' sign (b) inequality with '≥' sign
(c) inequality with '≤' sign (d) any or all of the above

2. The general linear programming problem is in standard form if
(a) the constraints are strict equations (b) the constraints are inequalities of ≤ type (c) the constraints are inequalities ≥ type (d) the decision variables are unrestricted in sign

3. Graphical method is useful to solve the LPP, when there are,
(a) only two decision variables (b) more than two decision variables
(c) objective function is non linear (d) decision variables are not positive

4. A necessary and sufficient condition for a basic feasible solution to a minimization LPP to be an optimum is that (for all j):
(a) $(z_j - c_j) \geq 0$ (b) $(z_j - c_j) \leq 0$ (c) $(z_j - c_j) = 0$ (d) $(z_j - c_j) > 0$
(e) $(z_j - c_j) < 0$

5. For maximization LPP, the objective function coefficient for an artificial variable is
(a) +M (b) −M (c) +1 (d) zero

6. If an optimum solution is degenerate, then
(a) the solution is infeasible (b) there are alternative optimum solutions (c) the solution is of no use to the decision maker
(d) none of the above

7. The solution to a transportation problem with m-sources and n-destination is feasible, if the number of allocations are
(a) $m+n-1$ (b) $m+n+1$ (c) $m+n$ (d) $m \times n$

8. The initial solution of a transportation problem obtained by
(a) north-west corner rule would invariably be optimum
(b) least cost method does not provide the least cost solution to a transportation problem
(c) VAM would invariably be very near to optimum solution
(d) MODI method is infeasible

9. The minimum number of lines covering all zeros in a reduced cost matrix of order n can be
(a) at the most n (b) at the least n (c) $n-1$ (d) $n+1$

10. When sum of gains of one player is equal to sum of losses to another player, the situation is known as
(a) Zero-sum game (b) fair game (c) Conflicting game
(d) negotiable game

11. When maximin and minimax values of the game are same, then
(a) there is a saddle point (b) solution does not exist
(c) Strategies are mixed (d) none of the above

12. A mixed strategy game can be solved by
(a) Matrix method (b) algebraic method
(c) graphical method (d) all of the above

13. What is the objective in solving problems with 2 jobs through k machines
(a) minimize total elapsed time (b) maximize total elapsed time
(c) maximize the idle time (d) none of the above.

14. No machine can process more than ____ operation at a time
(a) one (b) two (c) three (d) none

15. The time required to transfer jobs between machines is ____
(a) negligible (b) more significant (c) important
(d) None.

Part - B

II. Answer any two questions:- $2\times5=10$

16. Solve the following LPP by Graphical method

$\max\ z = x_1 + x_2$ Subject to the

Constraints $x_1 + x_2 \le 1$.

$-3x_1 + x_2 \ge 3$

$x_1 \ge 0,\ x_2 \ge 0$

17. Write the dual of the following primal of the LPP

$\min z = 4x_1 + 5x_2 - 3x_3$

Subject to $x_1 + x_2 + x_2 = 2$

$3x_1 + 5x_2 - 2x_3 \le 65$

$x_1 + 7x_2 + 4x_3 \ge 120$.

$x_1, x_2 \ge 0$ and x_3 is unrestricted.

18. Determine an initial basic feasible solution to the following transportation problem using NWCR method.

	D_1	D_2	D_3	D_4	Supply
	6	4	1	5	14
	8	9	2	7	16
	4	3	6	2	5
Required	6	10	15	4	35

19 Find the saddle point and the value of the game

	B_1	B_2	B_3	B_4
A_1	-5	4	1	20
A_2	5	5	4	6
A_3	3	-2	0	-5

20 Find the sequence that minimize the total elapsed time required to complete the following tasks on two machines

Task	A	B	C	D	E	F	G	H	I
machine - I	2	5	4	9	6	8	7	5	4
machine - II	6	8	7	4	3	9	3	8	11

Part - C

III. Answer All the questions: $5 \times 10 = 50$

21. (a) Using simplex method, solve LPP

$$\max z = x_1 + x_2 + 3x_3$$

Subject to
$$3x_1 + 2x_2 + x_3 \le 3$$
$$2x_1 + x_2 + 2x_3 \le 2$$
$$x_1, x_2, x_3 \ge 0$$

(or)

(b) Using simplex method, solve the LPP

$$\max z = 15x_1 + 6x_2 + 9x_3 + 2x_4$$

Subject to
$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

22 (a) Use two-phase method to solve

$$\max z = 5x_1 + 3x_2$$

Subject to
$$2x_1 + x_2 \le 1.$$
$$x_1 + 4x_2 \le 6$$
$$x_1, x_2 \ge 0.$$

(or)

(b) Solve by Big-M method

$$\max z = x_1 + 2x_2 + 3x_3 - x_4$$

Subject to
$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10.$$

23 (a) Solve the following transportation problem.

Destination

		P	Q	R	S	Supply
Origin	A	21	16	25	13	11
	B	17	18	14	23	13
	C	32	17	18	41	19
Demand		6	10	12	15	43

(or)

(b) Solve the transportation problem when the unit transportation costs, demands and supplies are as given below.

Destination

		D_1	D_2	D_3	D_4	Supply
Origin	O_1	6	1	9	3	70
	O_2	11	5	2	8	55
	O_3	10	12	41	7	70
Demand		85	35	50	45	

24. (a) Solve the following game and determine the value of the game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array}$$

(or)

(b) Solve the following 2 x 3 game graphically.

$$\begin{array}{cc} & \text{player } B \\ \text{player } A & \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix} \end{array}$$

25. (a) We have five jobs each of which must go through the machines A, B and C in the order ABC. Determine the sequence that will minimise the total elapsed time.

Job No	1	2	3	4	5
A	5	7	6	9	5
B	2	1	4	5	3
C	3	7	5	6	7

(or)

(b) Use graphical method to minimise the time needed to process the following jobs on the machines shown below. i.e) for each machine find the job which should be done first. Also calculate the total time needed to complete both the jobs.

Job 1	Sequence of m/c	A	B	C	D	E
	Time	2	3	4	6	2
Job 2	Sequence of m/c	C	A	D	E	B
	Time	4	5	3	2	6

HOD

A. Rosli
Subject Incharge
[A ROSALINE MARY]

... Arts & Science College for Women, Bargur

Cyclic Test - II - 2024.

Subject : Calculus of Variations & Integral Equations

Class : II - M.Sc Mathematics

Max Marks : 50

Hrs : 1.30 hrs

I. Answer All the Questions :

10×5 = 50

1. Find the shortest path from the point $A(-2,3)$ to the point $B(2,3)$ located in the region $y \le x^2$.
2. Find the extremals with corner points of the function $I[y(x)] = \int_{x_0}^{x_1} y'^2 (1-y')^2 \, dx$.
3. Find the extremum of the functional $I = \int_{x_1}^{x_2} (y'^2 + z'^2 + 2yz) \, dx$ with $y(0)=0$, $z(0)=0$ and the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$
4. Find the shortest distance between parabola $y = x^2$ and the straight line $x - y = 5$.
5. Using only the basic necessary condition $\delta I = 0$ find the curve on which an x_1 extremum of the function $I[y(x)] = \int_0^{x_1} \frac{(1+y')^2}{y} \, dx$ $y(0)=0$ can be archieved if the second boundary point (x_1, y_1) can move along the circumference $(x-9)^2 + y^2 = 9$.

Subject Incharge.

10 copies

GOVERNMENT ARTS AND SCIENCE COLLEGE FOR WOMEN
BARGUR- 635104.

M.SC MATHEMATICS MODEL EXAM

Subject : Calculus of Variation and Integral Equations Time: 3 Hours

Semester: IV Maximum:75 Marks

SECTION A — (15 × 1 = 15 Marks)

Answer ALL questions.

1. If a function $I[y(x)]$ attains a maximum on a curve $y = y_0(x)$ then at $y = y_0(x)$ $\delta I =$ ————————. (a) $-\infty$ (b) ∞ (c)0 (d)1
2. The function f(x) is decreasing around x_0 if $f'(x_0) = 0$ and $f''(x_0) > 0$ then x_0 is said to be -------- (a)Local minimum (b) Local maximum (c) increasing (d)decreasing
3. The Euler equation for the functional yields $I[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)dx$, $y(0) = 0, y(\pi/2) = 1$

   (a) $y'' + y = 0$ (b) $y'' - y = 0$ (c) $y'' - 2y = 0$ (d) $y'' + 2y = 0$

4. State Weirstrass-Erdmann corner conditions.
5. Write the transversality condition.
6. State orthogonality condition for the moving boundary.
7. A kernal K(x,t) is convolution if

   (a) $K(x,t) \neq \overline{K(x,t)}$ (b) $K(x,t) = \overline{K(x,t)}$ (c) $K(x,t) = K(x-t)$ (d) $K(x,t) = K(x+t)$
8. Two function $f_1(x)$ *and* $f_2(x)$ continuous on the interval [a, b] are said to be orthogonal on[a, b] if,

   (a) $\int_a^b f_1(x)f_2(x)\,dx \neq 1$ (b) $\int_a^b f_1(x)f_2(x)\,dx = 1$ (c) $\int_a^b f_1(x)f_2(x)\,dx = 0$ (d) $\int_a^b f_1(x)f_2(x)\,dx \neq 0$
9. The series is $u(x) = \lim_{n\to\infty} u_n(x) = f(x) + \sum_{m=1}^{\infty} \lambda^m \int_a^b k_m(x,t)f(t)\,dt$ known as ------series.

   (a) Abel (b) Neumann (c) Fisk (d) Dirchlet
10. The solution of the integral equation $u(x) = \frac{5x}{2} + \frac{1}{2}\int_0^1 xtu(t)\,dt$ is

    (a) u(x)=0 (b)u(x)=1 (c) u(x)=x (d) u(x)=2x+5.
11. Write the separable kernal of Fredhlom integral equations.
12. Two function $f_1(x)$ *and* $f_2(x)$ are said to be orthogonal

    (a) $(f_1, f_2) = \infty$ (b) $(f_1, f_2) = 1$ (c) $(f_1, f_2) \neq 0$ (d) $(f_1, f_2) = 0$
13. A kernal k(x,t) is said to be Hilbert -Schmidt kernal, if it is ________________ and square integrable (a)Symmetric (b) Skew symmetric(c) Skew Hemitian (d) Hermitian
14. Every symmetric kernal with a norm not equal to zero has at least ____eigen values.

    (a) One (b) two (c) three (d) four
15. Define Hilbert Space.

SECTION -B(2X5=10 Marks)

Answer any two questions:

16. Find the curve with fixed boundary points such that its rotation about the axis of abscissa given rise to a surface of revolution of minimum surface area.

17. Find the extremals with corner points of the functional $I[y(x)] = \int y'^{2}(1-y')^{2}\,dx$

18. Convert the following differential equation into integral equation with $y(0)=y'(0)=0$ $y'' + y = 0$

19. Solve $y(x) = 1 + \int_{1}^{x} y(t)\,dt$

20. State and prove Hilbert-Schmidt theorem

Answer all the questions: Section -C(5*10=50 Marks)

21. (a)Derive the Eulers equation of the variation problem. (OR)

    (b)State and prove Brachistochrone problem

22. (a) Using only the basic necessary condition *$\delta I=0$ find the curve on which an extremum of the functional* $I[y(x)] = \int_{0}^{x_1} \frac{\left(1+y'^{2}\right)^{\frac{1}{2}}}{y}\,dx,\ y(0)=0$ can be achieved if the second boundary point (x_1, y_1) can move along the circumference $(x-9)^2 + y^2 = 9$ (OR)

    (b)Find the shortest distance between the parabola $y=x^2$ and the straight in $x-y=5$.

23. (a) Show that the transformation is a $y(x) = \left(1+x^2\right)^{-3/2}$ solution of the integral equation

    $$y(x) = \frac{1}{1+x^2} - \int_{0}^{\infty} \frac{t}{\left(1+x^2\right)} y(t)\,dt. \qquad \text{(OR)}$$

    (b)Find the Eigen values and Eigen function of the homogeneous integral equation $u(x) = \lambda \int_{-1}^{1} \left(5xt^3 + 4x^2t + 3tx\right) u(t)\,dt$

24. (a)Find the resolvant kernal of $y(x) = f(x) + \lambda \int_{0}^{1} e^{t-x} y(t)\,dt$

    (OR)

    (b)Using the method of degenerate Kernals solve the integral equation

    $$u(x) - \lambda \int_{0}^{\pi} \cos\left(\log t^{x}\right) u(t)\,dt = 1$$

25. (a) Derive the solution of Voltera integral equation of the second kind by successive substations.

    (OR)

    (b)Solve the symmetric Fredhlom integral equation of the first kind

    $$f(x) = \int_{0}^{1} K(x,t)\,y(t)\,dt \quad \text{Where } K(x,t) = \begin{cases} x(1-t), & x<1 \\ t(1-x), & x>1 \end{cases}$$

S.No. 329

17PMA05

(For the candidates admitted from 2017 – 2018 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics

ALGEBRA

Time : Three hours

Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Normalizer of a in G.
2. Define equivalence relation in group.
3. Define internal direct product.
4. Define invariants.
5. Define free module.
6. Define epimorphism.
7. Define Galois group.

8. Define normal extension of field F.

9. Define finite field.

10. State Frobenius theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) If $o(G) = p^2$ where p is a prime number then prove that the group G is abelian.

Or

(b) Show that $n(k) = 1 + p + ... + p^{k+1}$.

12. (a) Let G be a group and suppose that G is the internal direct product of $N_1, ..., N_n$. Let $T = N_1 \times N_2 \times ... \times N_n$. Then prove that G and T are isomorphic.

Or

(b) If G and G are isomorphic abelian group then prove that for every integer s, $G(s)$ and $G'(s)$ are isomorphic.

13. (a) State and prove homomorphism theorem.

Or

(b) If A is a commutative ring and F is a free A-module then show that any two free basis of F have the same Cardinal number.

14. (a) Show that S_n is not solvable for $n \geq 5$.

Or

(b) Show that that the fixed field of G is a subfield of K.

15. (a) State and prove Jacobson theorem.

Or

(b) Prove that for every prime number p and every positive integer m then prove there exists a field having p^m elements.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Cauchy theorem.

17. Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

18. State and prove Second isomorphism theorem.

19. If $p(x) \in F[x]$ is solvable by radicals over F, then show that the Galois group over F of $p(x)$ is a solvable group.

20. State and prove Wedderburn's theorem.

18. Discuss Hamilton's principal function.

19. State and prove STACKEL's THEOREM.

20. Explain differential form of Canonical Transformation.

S.No. 309 **17PMA03**

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics

MECHANICS

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Degree of Freedom.

2. State the principle of virtual work.

3. Define Routhian function.

4. Write down the standard form of Lagrange's equation for a nouholonomic system.

5. State the Brachistochrofle problem.

6. Write the Jacobins form of the principle of least action.

7. Write the modified Hamilton-Jacobi equation.

8. State Stackel's theorem.

9. Define Bilinear covariant.

10. Define Homogeneous Canonical Transformations.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Derive the principle of conservation of energy.

Or

(b) Explain D'Alembert's principle.

12. (a) Derive the expression for K.E as, $T=\frac{1}{2}\sum_{k=1}^{3n} m_k\left[\sum_{i=1}^{n}\frac{\partial x_k}{\partial q_i}q_i+\frac{\partial x_k}{\partial t}\right]$ for $x_1, x_2, \ldots\ldots, x_{3n}$.

Or

(b) Write a short note on Ignorable coordinates.

13. (a) State Modified Hamilton's principle.

Or

(b) Find the stationary value of the function $f = z$ subject to the constraints $\varphi_1 = x^2 + y^2 + z^2 - 4 = 0$.

14. (a) With notations, derive $H\left[q, \frac{\partial w}{\partial q}\right] = \alpha_n$.

Or

(b) Explain the Pfaffian Differential Forms.

15. (a) Explain the Bilinear covariant.

Or

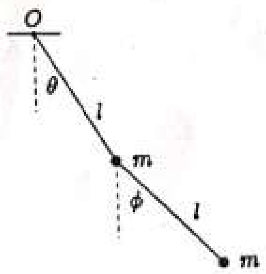
(b) Derive Jacobi identity property.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Write the Rotational Kinetic Energy in the form of $T_{rot} = \frac{1}{2}\omega^T I\omega$.

17. A double pendulum consists of two particles suspended by massless rods, as shown in figure. Assuming that all motion takes place in a vertical plane, find the differential equations of motion.



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S.No. 224

21PMA07

(For the candidates admitted from 2021–2022 onwards)

M.Sc. DEGREE EXAMINATION, JUNE 2022

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (15 × 1 = 15 marks)

Answer ALL the questions.

$1+\alpha^2 = \frac{v^2}{c^2}$

1. If $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, then

$\alpha^2 = \frac{v^2}{c^2} - 1$

 (a) $\alpha^2 + \frac{c^2}{v^2} = 1$ (b) $\alpha^2 - \frac{c^2}{v^2} = 1$

 (c) $\alpha^2 + \frac{v^2}{c^2} = 1$ (d) $\alpha^2 - \frac{v^2}{c^2} = 1$

2. The solution of $(\beta D' + \gamma)z = 0$ is

 (a) $\phi(\beta x - \gamma y)\exp\left(\frac{\gamma}{\beta}y\right)$

 (b) $\phi(-\beta x + \gamma y)\exp\left(-\frac{\gamma}{\beta}y\right)$

 (c) $\phi(\beta x)\exp\left(\frac{\gamma}{\beta}y\right)$

 (d) $\phi(\beta x)\exp\left(-\frac{\gamma}{\beta}y\right)$

3. The PDE $\sin^2 x u_{xx} + \sin(2x)u_{xy} - \cos^2 x u_{yy} = x$ is

 (a) Parabolic (b) Elliptic

 (c) Hyperbolic (d) None of these

4. Which of the following is elliptic?

 (a) Wave equation (b) Diffusion equation

 (c) Laplace equation (d) Transport equation

5. One of the possible solution of Laplace equation $\nabla^2 u = 0$ is

 (a) $(A\cos px + B\sin px)\left(Ce^{py^2} + De^{-py^2}\right)$

 (b) $(A\cos px + B\sin px)\left(Ce^{py} + De^{-py^2}\right)$

 (c) $(A\cos px + B\sin px)\left(Ce^{py^2} + De^{-py}\right)$

 (d) None of these